ΕΝΔΕΙΚΤΙΚΕΣ ΑΠΑΝΤΗΣΕΙΣ

ΜΑΘΗΜΑΤΙΚΑ ΓΕΝΙΚΗΣ

ΠΑΝΕΛΛΗΝΙΕΣ 2015

ΘΕΜΑ Α

Α1. Απόδειξη σχολ. Βιβλίου σελ. 31

Α2. Ορισμός σχολ. Βιβλίου σελ 22

Α3. Ορισμός σχολ. Βιβλίου σελ. 86-87

Α4. α. Λ β. Σ γ. Λ δ. Λ ε. Σ

ΘΕΜΑ Β

Β1.

$$(3x-1)(8x^2-6x+1)=0$$

$$3x-1=0 \Leftrightarrow x=\frac{1}{3}$$

ή

$$8x^2-6x+1=0$$

$$\Delta=36-32=4$$

$$x_{1,2}=\frac{6\pm 2}{16}=\begin{cases}\frac{8}{16}=\frac{2}{4}=\frac{1}{2}\\ \frac{6-2}{16}=\frac{4}{16}=\frac{1}{4}\end{cases}$$

Ισχύει ότι:

$A\cap B\subseteq A\subseteq A\cup B$ οπότε

$P(A\cap B)\leq P(A)\leq P(A\cup B)$

Άρα $P(A\cap B)=\frac{1}{4}$, $P(A)=\frac{1}{3}$ $\kappa\alpha\iota$ $P(A\cup B)=\frac{1}{2}$

Β2.

$\mathrm{A}' - \mathrm{B}' = \mathrm{A}' \cap (\mathrm{B}')' = \mathrm{A}' \cap \mathrm{B}$ =Β-Α

ή

$$
\begin{aligned}
P(\mathrm{A}' - \mathrm{B}') &= P(\mathrm{A}') - P(\mathrm{A}' \cap \mathrm{B}') = \\
&= 1 - P(\mathrm{A}) - P(\mathrm{A} \cup \mathrm{B})' = \\
&= 1 - P(\mathrm{A}) - (1 - P(\mathrm{A} \cup \mathrm{B})) = \\
&= 1 - P(\mathrm{A}) - 1 + P(\mathrm{A} \cup \mathrm{B}) = \\
&= P(\mathrm{A} \cup \mathrm{B})) - P(\mathrm{A}) = \\
&= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}
\end{aligned}
$$

$$P(\Delta) = P(\mathrm{A} \cap \mathrm{B})' = 1 - P(\mathrm{A} \cap \mathrm{B}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Β3.

$$\mathrm{E} = (\mathrm{A} - \mathrm{B}) \cup (\mathrm{B} - \mathrm{A})$$

$$P(\mathrm{E}) = P(\mathrm{A} - \mathrm{B}) + P(\mathrm{B} - \mathrm{A}) = P(\mathrm{A}) - P(\mathrm{A} \cap \mathrm{B}) + P(\mathrm{B}) - P(\mathrm{A} \cap \mathrm{B}) =$$

$$P(\mathrm{A} \cup \mathrm{B}) - P(\mathrm{A} \cap \mathrm{B}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

Β4.

$$9x^2 - 3x - 2 = 0$$

$$\Delta = 9 - 4 \cdot 9 \cdot (-2) = 9 + 72 = 81$$

$$x_{1,2} = \frac{3 \pm 9}{18} = \left\langle \begin{array}{l} \dfrac{12}{18} = \dfrac{2}{3} \\[2ex] -\dfrac{6}{18} \quad \text{ΑΠΟΡ.} \end{array} \right.$$

Άρα $P(\Gamma) = \dfrac{2}{3}$

- $P(\mathrm{A} \cup \mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A} \cap \mathrm{B})$

  οπότε

  $$\frac{1}{2} = \frac{1}{3} + P(\mathrm{B}) - \frac{1}{4} \Rightarrow \frac{1}{2} - \frac{1}{3} + \frac{1}{4} = P(\mathrm{B}) \Rightarrow \frac{6}{12} - \frac{4}{12} + \frac{3}{12} = P(\mathrm{B}) \Rightarrow P(\mathrm{B}) = \frac{5}{12}$$

Έστω ότι Β και Γ είναι ασυμβίβαστα

οπότε $P(B \cup \Gamma) = \frac{2}{3} + \frac{5}{12} = \frac{8+5}{12} = \frac{13}{12} > 1$

Άρα Β και Γ δεν είναι ασυμβίβαστα.

ΘΕΜΑ Γ

Γ1.

Κλάσεις	f_i%	x_i	f_i	$x_i f_i$	$(x_i - \bar{x})^2$
[8, 10)	10	0,1	9	0,9	$(9-14)^2 = 25$
[10, 12)	10	0,1	11	$f_2 \cdot 11$	$(11-14)^2 = 9$
[12, 14)	30	0,3	13	3,9	$(13-14)^2 = 1$
[14, 16)	20	0,2	15	$15 \cdot f_4$	$(15-14)^2 = 1$
[16, 18)	30	0,3	17	5,1	$(17-14)^2 = 9$
ΣΥΝΟΛΟ	100	1		14	

f_1% =10

f_5% =30

α_3=108.

Ισχύει ότι $a_3 = 360 \cdot f_3 \Rightarrow 108 = 360 \cdot f_3 \Rightarrow f_3 = \frac{108}{360} = 0{,}3$

$\bar{x} = 0{,}9 + 11 f_2 + 3{,}9 + 15 f_4 + 5{,}1 = 9{,}9 + 11 f_2 + 15 f_4 \Rightarrow 14 = 9{,}9 + 11 f_2 + 15 f_4 \Rightarrow$

$\Rightarrow 14 - 9{,}9 = 11 f_2 + 15 f_4$

$4{,}1 = 11 f_2 + 15 f_4$ (1)

$f_2 + f_4 = 1 - (0{,}1 + 0{,}6) = 1 - 0{,}7 = 0{,}3$ (2)

Από (1) και (2) $f_2 = 0{,}1$ και $f_4 = 0{,}2$

Γ2.

$$S^2 = \frac{1}{\nu}\sum_{i=1}^{\kappa}\left(x_i - \bar{x}\right)^2 \cdot \nu_i = \sum_{i=1}^{\kappa}\left(x_i - \bar{x}\right)^2 f_i$$

$$S^2 = 6{,}6 \qquad\qquad S = \sqrt{6{,}6} = 2{,}57$$

$$CV = \frac{2{,}57}{14} = 18{,}3\% > 10\%$$ **όχι ομοιογενές**

Γ3.

	K_i	f_i	ν_i	$k_i\nu_i$
[8, 10)	9	0,1	0,1ν	0,9ν
[10, 12)	11	0,1	0,1ν	1,1ν
[12, 14)	13	0,3	0,3ν	3,9ν
[14, 16)	15	0,2	0,2ν	3ν
[16, 18)	17	0,3	0,3ν	
ΣΥΝΟΛΟ		1		

$$\sum_{i=1}^{4} x_i \nu_i = 1780$$

$$9 \cdot 0{,}1\nu + 1{,}1\nu + 3{,}9\nu + 3\nu = 1780$$

$$0{,}9\nu + 1{,}1\nu + 3{,}9\nu + 3\nu = 1780$$

$$8{,}9\nu = 1780$$

$$\nu = \frac{1780}{8{,}9} = 200$$

Γ4.

$$\beta_i = \frac{a_i}{S_a} - \frac{\bar{a}}{S_a} = a_i \cdot \frac{1}{S_a} - \frac{1}{CV_a}$$

$$\bar{\beta} = \bar{a} \cdot \frac{1}{S_a} - \frac{1}{CV_a} = \frac{1}{CV_a} - \frac{1}{CV_a} = 0$$

$$S_\beta = \left|\frac{1}{Sa}\right| \cdot Sa \overset{Sa>0}{=} 1$$

ΘΕΜΑ Δ

Δ1.

$E = x \cdot y$ με $y = \text{ΒΓ}$

Θεωρώ Δ μέσο της ΒΓ και εφαρμόζω πυθαγόρειο θεώρημα στο ΟΒΓ

Είναι $OB^2 = O\Delta^2 + \Delta B^2$

$$\rho^2 = (\frac{x}{2})^2 + (\frac{y}{2})^2 \Rightarrow 25 = \frac{x^2}{4} + \frac{y^2}{4}$$

$$\left.\begin{array}{l} y^2 = 100 - x^2 \\ \kappa\alpha\iota\ \alpha\varphi o\acute{\upsilon}\ y > 0 \end{array}\right\} \Rightarrow y = \sqrt{100 - x^2}$$

$E(x) = x \cdot \sqrt{100 - x^2}$ με

$$\left.\begin{array}{l} x > 0 \\ y > 0 \Leftrightarrow \sqrt{100 - x^2} > 0 \\ 100 - x^2 > 0 \\ x^2 < 100 \\ |x| < 10 \\ -10 < x < 10 \end{array}\right\} \Rightarrow x \in (0,10)$$

Δ2.

$$f'(x) = \sqrt{100 - x^2} + x \cdot \frac{1 \cdot (-2x)}{2 \cdot \sqrt{100 - x^2}} = \sqrt{100 - x^2} - \frac{x^2}{\sqrt{100 - x^2}} = \frac{100 - x^2 - x^2}{\sqrt{100 - x^2}} = \frac{2 \cdot (50 - x^2)}{\sqrt{100 - x^2}}$$

x	0		$5\sqrt{2}$		10
$f'(x)$		+	0	-	
$f(x)$		↗	max	↘	

$$\left.\begin{array}{l} \gamma\iota\alpha\ \ x = 5\sqrt{2} \\ y = \sqrt{100 - (5\sqrt{2})^2} = \sqrt{50} = 5\sqrt{2} \end{array}\right\} \Rightarrow y = x = 5\sqrt{2}$$

Άρα το ορθογώνιο είναι τετράγωνο

Δ3.

$$\lim_{x\to 0}\frac{f(1+x)-\sqrt{99}}{98x}=\frac{1}{98}\lim_{x\to 0}\frac{f(x+1)-f(1)}{x+1-1}=$$

$$\frac{1}{98}\cdot f'(1)=\frac{1}{98}\cdot\frac{2\cdot(50-1)}{\sqrt{100-1}}=\frac{1}{98}\cdot\frac{2\cdot 49}{\sqrt{99}}=\frac{1}{\sqrt{99}}=\frac{\sqrt{99}}{99}$$

2^{ος} τρόπος

Δ3.

$$\lim_{x\to 0}\frac{f(1+x)-\sqrt{99}}{98\cdot x}=\lim_{x\to 0}\frac{(1+x)\sqrt{100-(1+x)^2}-\sqrt{99}}{98\cdot x}=\lim_{x\to 0}\frac{(1+x)\sqrt{99-2x-x^2}-\sqrt{99}}{98\cdot x}=$$

$$=\lim_{x\to 0}\frac{\left[(1+x)\sqrt{99-2x-x^2}-\sqrt{99}\right]\left[(1+x)\sqrt{99-2x-x^2}+\sqrt{99}\right]}{98\cdot x\left[(x+1)\sqrt{99-2x-x^2}+\sqrt{99}\right]}=$$

$$=\lim_{x\to 0}\frac{(1+x)^2\left(99-2x-x^2\right)-99}{98\cdot x\left[(x+1)\sqrt{99-2x-x^2}+\sqrt{99}\right]}$$

$$=\lim_{x\to 0}\frac{-x^4-2x^3+94x^2+196x}{98x\left[(x+1)\sqrt{99-2x-x^2}+\sqrt{99}\right]}=\lim_{x\to 0}\frac{x\left(-x^3-2x^2+94x+196\right)}{98x\left[(x+1)\sqrt{99-2x-x^2}+\sqrt{99}\right]}$$

$$\lim_{x\to 0}\frac{-x^3-2x^2+94x+196}{98\left[(x+1)\sqrt{99-2x-x^2}+\sqrt{99}\right]}=\frac{196}{2\sqrt{99}}=\frac{1}{\sqrt{99}}=\frac{\sqrt{99}}{99}$$

Δ4.

$(0\,,\,1]\subseteq[0\,,\,5\sqrt{2}]$, $0<P(A)\le 1$, $0<P(A-B)\le 1$

Η f είναι γνησίως αύξουσα, οπότε αφού $A-B\subseteq A$, θα ισχύει $P(A-B)\le P(A)$ και

$$f(P(A-B))\le f(P(A))\Rightarrow P(A-B)\cdot\sqrt{100-P^2(A-B)}\le P(A)\cdot\sqrt{100-P^2(A)}\Rightarrow$$

$$\Rightarrow\frac{P(A-B)}{\sqrt{100-P^2(A)}}\le\frac{P(A)}{\sqrt{100-P^2(A-B)}}\overset{f\ \gamma\nu.}{\underset{\alpha\acute{\upsilon}\xi o\upsilon\sigma\alpha}{\Rightarrow}}f\left(\frac{P(A-B)}{\sqrt{100-P^2(A)}}\right)\le f\left(\frac{P(A)}{\sqrt{100-P^2(A-B)}}\right)$$

Αφού

$$0<P(A-B)\le 1$$
$$0<P^2(A-B)\le 1$$
$$-1\le -P^2(A-B)<0$$
$$99\le 100-P^2(A-B)<100$$
$$\sqrt{99}\le \sqrt{100-P^2(A-B)}<10$$
$$\frac{1}{10}<\frac{1}{\sqrt{100-P^2(A-B)}}\le \frac{1}{\sqrt{99}} \quad (1)$$
$$\kappa\alpha\iota \;\; 0<P(A)\le 1 \qquad (2)$$

πολλαπλασιάζω κατά μέλη (1) *και* (2)

και έχω

$$0<\frac{P(A)}{\sqrt{100-P^2(A-B)}}\le \frac{1}{\sqrt{99}}$$

$$0<P(A)\le 1$$
$$0<P^2(A)\le 1$$
$$-1\le -P^2(A)<0$$
$$99\le 100-P^2(A)<100$$
$$\sqrt{99}\le \sqrt{100-P^2(A-B)}<10$$
$$\frac{1}{10}<\frac{1}{\sqrt{100-P^2(A)}}\le \frac{1}{\sqrt{99}} \quad (3)$$
$$\kappa\alpha\iota \;\; 0<P(A-B)\le 1 \qquad (4)$$

πολλαπλασιάζω κατά μέλη (3) *και* (4)

και έχω

$$0<\frac{P(A-B)}{\sqrt{100-P^2(A)}}\le \frac{1}{\sqrt{99}}$$