

**ΜΑΘΗΜΑΤΙΚΑ ΚΑΙ ΣΤΟΙΧΕΙΑ ΣΤΑΤΙΣΤΙΚΗΣ**  
**ΓΕΝΙΚΗΣ ΠΑΙΔΕΙΑΣ**  
**ΠΑΝΕΛΛΗΝΙΕΣ 2010**  
**ΕΝΔΕΙΚΤΙΚΕΣ ΑΠΑΝΤΗΣΕΙΣ**

**ΘΕΜΑ Α**

**A1:** θεωρία σελ 93.

**A2:** θεωρία σελ. 86-87.

**A3:** θεωρία σελ. 140.

**A4:** α) Σ

β) Λ

γ) Σ

δ) Λ

ε) Λ

**ΘΕΜΑ Β**

**B1**

$$\lim_{x \rightarrow 1} \frac{2\sqrt{x^2 - x + 1} - 1 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{2(\sqrt{x^2 - x + 1} - 1)}{x - 1} \stackrel{0}{=} \lim_{x \rightarrow 1} \frac{2(\sqrt{x^2 - x + 1} - 1)(\sqrt{x^2 - x + 1} + 1)}{(x - 1)(\sqrt{x^2 - x + 1} + 1)} =$$
$$\lim_{x \rightarrow 1} \frac{2(x^2 - x + 1 - 1)}{(x - 1)(\sqrt{x^2 - x + 1} + 1)} = \lim_{x \rightarrow 1} \frac{2x(x - 1)}{(x - 1)(\sqrt{x^2 - x + 1} + 1)} = \frac{2 \cdot 1}{2} = 1$$

**B2**

$$f'(x) = 2 \cdot \frac{1}{2\sqrt{x^2 - x + 1}} \cdot (x^2 - x + 1)' = \frac{2x - 1}{\sqrt{x^2 - x + 1}}$$

Άρα

$$\lambda = f'(0) = \frac{2 \cdot 0 - 1}{\sqrt{1}} = -1$$

**B3**

$$\varepsilon\phi\omega = \lambda \text{ οπότε } \varepsilon\phi\omega = -1 \Leftrightarrow \omega = \frac{3\pi}{4}$$

**ΘΕΜΑ Γ**

	$x_i$
$[0, 4)$	
$[4, 8)$	6

**Γ1.**  $\frac{3C}{2} = 6 \Leftrightarrow 3C = 12 \Leftrightarrow C = 4$

**Γ2**

	$x_i$	$v_i$	$x_i v_i$	$x_i^2 v_i$
$[0,4)$	2	20	40	80
$[4,8)$	6	40	240	1440
$[8,12)$	10	45	450	4500
$[12,16)$	14	30	240	5880
$[16,20)$	18	25	450	8100
<b>ΣΥΝΟΛΟ</b>		160	1600	20.000

$$\bar{x} = \frac{1}{v} \cdot \sum_{i=1}^k x_i v_i = \frac{1600}{160} = 10 \text{ κιλά}$$

$$s^2 = \frac{1}{v} \cdot \left\{ \sum (x_i)^2 v_i - \frac{(\sum x_i v_i)^2}{v} \right\} = \frac{1}{160} \cdot \left\{ 20.000 - \frac{(1600)^2}{160} \right\} = \frac{1}{160} \cdot 4000 = 25 \text{ κιλά}^2$$

οπότε  $s = \sqrt{25} = 5$  κιλά.

**Γ3.**  $cv = \frac{s}{\bar{x}} = \frac{5}{10} = \frac{1}{2} \rightarrow 50\%$

**Γ4.**  $\frac{1}{4} \cdot 40 + 45 + \frac{1}{2} \cdot 30 = 10 + 15 + 45 = 70$  άτομα

$$P(A) = \frac{70}{160} = 0,4375$$

**ΘΕΜΑ Δ**

$$f(x) = \ln(x - P(A)) - \frac{1}{2}(x - P(A))^2 + P(B), x > P(A)$$

**Δ1.**

$$f'(x) = \frac{1}{x - P(A)} - \frac{1}{2} \cdot 2(x - P(A)) = \frac{1}{x - P(A)} - (x - P(A)) =$$

$$= \frac{1 - [x - P(A)]^2}{x - P(A)} \left. \begin{array}{l} x - P(A) = 1 \Rightarrow x = 1 + P(A) \\ f'(x) = 0 \end{array} \right\}$$

η f ↑ στο  $(P(A), 1 + P(A)]$ η f ↓ στο  $[1 + P(A), +\infty)$ Το  $\left(1 + P(A), P(B) - \frac{1}{2}\right)$  Τ.Μ**Δ2.**

$$x_0 = \frac{5}{3} \text{ σημείο Τ.Α} \Rightarrow 1 + P(A) = \frac{5}{3} \Rightarrow P(A) = \frac{2}{3}$$

$$f(x_0) = 0 \Rightarrow f\left(\frac{5}{3}\right) = 0$$

$$\ln\left(\frac{5}{3} - P(A)\right) - \frac{1}{2}\left(\frac{5}{3} - P(A)\right)^2 + P(B)$$

$$\ln 1 - \frac{1}{2}1^2 + P(B) = P(B) - \frac{1}{2} = 0 \Rightarrow P(B) = \frac{1}{2}$$

**Δ3.**

$$P(A \cup B) = \frac{5}{6}$$

$$\Delta = (A \cap B)' \Rightarrow P(\Delta) = P(A \cap B)' = 1 - P(A \cap B) = 1 - (P(A) + P(B) - P(A \cup B))$$

$$= 1 - P(A) - P(B) + P(A \cup B) = 1 - \frac{2}{3} - \frac{1}{2} + \frac{5}{6} = \frac{6}{6} - \frac{4}{6} - \frac{3}{6} + \frac{5}{6} = \frac{4}{6} = \frac{2}{3}$$

**Δ4.**

$$E = (A - B) \cup (B - A)$$

$$P(E) = P(A \cup B) - P(A \cap B) = \frac{5}{6} - \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$$